SOLUTION OF HYPERBOLIC HEAT- AND MASS-TRANSFER EQUATIONS

N. I. Gamayunov

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A method is presented for the solution of heat- and mass-transfer equations (1)–(2) for generalized conditions of the second kind. The solutions are compared with the earlier derived analogous solutions, without consideration of the Fourier relaxation criterion.

Reference [1] presents new phenomenological heat- and mass-transfer equations of the parabolic-hyperbolic type in which the finite rate of moisture propagation in capillary-porous bodies is taken into consideration:

$$\frac{\partial T}{\partial \mathrm{Fo}} = \nabla^2 T - \varepsilon \,\mathrm{Ko} \frac{\partial \Theta}{\partial \mathrm{Fo}}, \tag{1}$$

$$\mathrm{Fo}_{rm} \frac{\partial^2 \Theta}{\partial \mathrm{Fo}^2} + \frac{\partial \Theta}{\partial \mathrm{Fo}} = \mathrm{Lu}\, \nabla^2 \Theta - \mathrm{Lu}\,\mathrm{Pn}\, \nabla^2 T, \tag{2}$$

where $\mathrm{Fo}_{rm} = a\tau_{rm}/R^2$ is the relaxation mass-transfer Fourier criterion; τ_{rm} is the relaxation time; $\nabla^2 = \partial^2/\partial X^2 + (m-1)/X\,(\partial/\partial X)$ $(0 \le X \le 1)$ is the Laplace operator. The values of m for classical bodies are presented in the table.

Let us solve system (1)–(2) for generalized boundary conditions of the second kind [2]:

$$-\frac{\partial T(1, \mathrm{Fo})}{\partial X} + \mathrm{Ki}_q(\mathrm{Fo}) - (1-\varepsilon)\,\mathrm{Lu}\,\mathrm{Ko}\,\mathrm{Ki}_m(\mathrm{Fo}) = 0, \tag{3}$$

$$-\frac{\partial \Theta(1, \mathrm{Fo})}{\partial X} + \mathrm{Pn}\frac{\partial T(1, \mathrm{Fo})}{\partial X} + \mathrm{Ki}_m(\mathrm{Fo}) = 0, \tag{4}$$

$$\frac{\partial T(0, \mathrm{Fo})}{\partial X} = 0; \quad \frac{\partial \Theta(0, \mathrm{Fo})}{\partial X} = 0 \tag{5}$$

and initial conditions

$$T(X, 0) = T_0(X); \ \Theta(X, 0) = \Theta_0(X); \quad \frac{\partial \Theta(X, 0)}{\partial \mathrm{Fo}} = \Theta_1(X). \tag{6}$$

Applying the final integral transformation [2, 3] to (1) and (2) with the kernel $k(\mu_n X)$ (table), and then the integral Laplace transform over the variable Fo [2, 4, 5], we find

$$T_{kL} = \sum_{j=1}^{2} \delta_{1j}\Delta^{-1}\Phi_{jL}, \tag{7}$$

$$\Theta_{kL} = \sum_{j=1}^{2} \delta_{2j}\Delta^{-1}\Phi_{jL}, \tag{8}$$

where

$$\delta_{11} = \mathrm{Fo}_{rm}\,p^2 + p + \mathrm{Lu}\,\mu_n^2; \quad \delta_{12} = -p\,\varepsilon\,\mathrm{Ko}; \tag{9}$$

$$\delta_{21} = \mathrm{Lu}\,\mathrm{Pn}\,\mu_n^2; \quad \delta_{22} = p + \mu_n^2; \tag{10}$$

$$\Delta = \delta_{11}\delta_{22} - \delta_{12}\delta_{21} = \mathrm{Fo}_{rm}(p-p_1)(p-p_2)(p-p_3). \tag{11}$$

The roots p_i (i = 1, 2, 3) are found from the equation

$$p^3 + (\mu_n^2 + \mathrm{Fo}_{rm}^{-1})\,p^2 + (1 + \mathrm{Lu} + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn}\,\mathrm{Lu}) \times \mathrm{Fo}_{rm}^{-1}\,\mu_n^2\,p + \mathrm{Lu}\,\mathrm{Fo}_{rm}^{-1}\,\mu_n^4 = 0. \tag{12}$$

From the Kardano formula

$$p_1 = a_1 + b_1 - \frac{\alpha}{3},$$

$$p_{2,3} = -\frac{1}{2}(a_1 + b_1) - \frac{\alpha}{3} \pm i\frac{\sqrt{3}}{2}(a_1 - b_1),$$

where

$$a_1 = \sqrt[3]{-\frac{u}{2} + \sqrt{\frac{u^2}{4} + \frac{v^3}{27}}};$$

$$b_1 = \sqrt[3]{-\frac{u}{2} - \sqrt{\frac{u^2}{4} + \frac{v^3}{27}}};$$

$$u = \frac{2\alpha^3}{27} - \frac{\alpha\beta}{3} + \gamma; \quad v = -\frac{\alpha^2}{3} + \beta,$$

with $a_1 b_1 = -v/3$;

$$\alpha = \mu_n^2 + \mathrm{Fo}_{rm}^{-1} = -(p_1 + p_2 + p_3); \tag{13}$$

$$\beta = (1 + \mathrm{Lu} + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn}\,\mathrm{Lu})\,\mathrm{Fo}_{rm}^{-1}\,\mu_n^2 = p_1p_2 + p_1p_3 + p_2p_3; \tag{14}$$

$$\gamma = \mathrm{Lu}\,\mathrm{Fo}_{rm}^{-1}\,\mu_n^4 = -p_1p_2p_3. \tag{15}$$

The validity of Eqs. (13)–(15) follows from the theory of polynomials [6]. The convergence of series (21) and (22) presented below imposes the condition that all the roots p_i be negative. In particular, this follows from (13)–(15), since α, β, and γ are quantities that are always positive. The roots p_{in} are calculated from the above-cited formulas, but into these we must successively substitute the values of $\mu_1, \mu_2, \ldots, \mu_n$ $(n = 1, 2, \ldots, \infty)$.

If the discriminant $D = u^2/4 + v^3/27 < 0$, all roots are real and different; if $D = 0$, then p_1 and $p_2 = p_3$ are real numbers; if $D > 0$, p_1 is a real root and p_2 and p_3 are conjugate complex roots.

The reconversion for the variable X is accomplished with the formula [2–4]

$$\varphi(X, \mathrm{Fo}) = m\,[\varphi(0, \mathrm{Fo})]_k + 2\sum_{n=1}^{\infty} k_1(\mu_n)\,k(\mu_n X)\,[\varphi(\mu_n, \mathrm{Fo})]_k, \tag{16}$$

where the values of m, $k_1(\mu_n)$, and $k(\mu_n X)$ for classical bodies are given in the table.

To find $[\varphi(0, \mathrm{Fo})]_k$, i.e., $T_k(0, \mathrm{Fo})$ and $\Theta_k(0, \mathrm{Fo})$, it must be assumed in Eq. (12) that $\mu_n = 0$, so that we have

$$p^3 + p^2 \mathrm{Fo}_{rm}^{-1} = 0, \quad (17)$$

whence $p_1 = -\mathrm{Fo}_{rm}^{-1}$ and the double root $p_2 = p_3 = 0$. Using the formula

$$[\varphi(0, \mathrm{Fo})]_k = \lim_{p \to \mathrm{Fo}_{rm}^{-1}} \frac{\varphi(p)}{\psi_1(p)} \exp p\,\mathrm{Fo} + \lim_{p \to 0} \left[\mathrm{Fo} \exp p\,\mathrm{Fo} \frac{\varphi(p)}{\psi_2(p)} + \exp p\,\mathrm{Fo} \frac{\varphi'(p)}{\psi_2(p)} - \exp p\,\mathrm{Fo} \frac{\varphi(p)\psi_2'(p)}{[\psi_2(p)]^2} \right], \quad (18)$$

where

$$\psi_1 = \mathrm{Fo}_{rm} p^2; \quad \psi_2 = \mathrm{Fo}_{rm}(p + \mathrm{Fo}_{rm}^{-1}).$$

Since $\varphi(p) = \delta_{lj}\Phi_j(l, j = 1, 2)$, as follows from (9)–(10),

$$\lim_{\mu_n,\, p \to 0} \delta_{lj}\Phi_j = 0, \text{ while } \lim_{\mu_n,\, p \to 0} \varphi'(p) = \lim_{\mu_n,\, p \to 0} \delta_{lj}'\Phi_j.$$

After calculation and transformation we have the final solution for $D < 0$ and $p_1 \neq p_2 \neq p_3 < 0$:

$$T(X, \mathrm{Fo}) = \sum_{j=1}^{2} \sum_{i=1}^{3} \Phi_{1ji}, \quad (19)$$

$$\Theta(X, \mathrm{Fo}) = \sum_{j=1}^{2} \sum_{i=1}^{3} \Phi_{2ji}, \quad (20)$$

where

$$\Phi_{l1} = \sum_{i=1}^{3} \Phi_{l1i} = \nu_{l12} m \int_0^1 X^{m-1} [T_0(X) + \varepsilon \mathrm{Ko}\,\Theta_0(X)]\, dX + 2 \sum_{n=1}^{\infty} \sum_{i=1}^{3} \delta_{l1in} \Delta_{in}^{-1} k_1(\mu_n) k(\mu_n X) \exp(p_{in}\mathrm{Fo}) \times \int_0^1 [T_0(X) + \varepsilon \mathrm{Ko}\,\Theta_0(X)] \bar{k}(\mu_n X)\, dX + \nu_{l12} m \int_0^{\mathrm{Fo}} \Psi_1(\mathrm{Fo}^*)\, d\mathrm{Fo}^* + 2 \sum_{n=1}^{\infty} \sum_{i=1}^{3} \delta_{l1in} \Delta_{in}^{-1} k_2(\mu_n) k(\mu_n X) \times \int_0^{\mathrm{Fo}} \Psi_1(\mathrm{Fo}^*) \exp[p_{in}(\mathrm{Fo} - \mathrm{Fo}^*)]\, d\mathrm{Fo}^*; \quad (21)$$

$$\Phi_{l2} = \sum_{i=1}^{3} \Phi_{l2i} = \nu_{l21} \left\{ \mathrm{Fo}_{rm} \exp(-\mathrm{Fo}/\mathrm{Fo}_{rm})\, m \int_0^1 X^{m-1} \Theta_1(X)\, dX + m \int_0^{\mathrm{Fo}} \Psi_2(\mathrm{Fo}^*) \exp[-(\mathrm{Fo} - \mathrm{Fo}^*)/\mathrm{Fo}_{rm}]\, d\mathrm{Fo}^* \right\} + \nu_{l22} \left\{ m \int_0^1 X^{m-1} [\mathrm{Fo}_{rm} \Theta_1(X) + \Theta_0(X)]\, dX + m \int_0^{\mathrm{Fo}} \Psi_2(\mathrm{Fo}^*)\, d\mathrm{Fo}^* \right\} + 2 \sum_{n=1}^{\infty} \sum_{i=1}^{3} \delta_{l2in} \Delta_{in}^{-1} k(\mu_n X) \times \left\{ \exp(p_{in}\mathrm{Fo})\, k_1(\mu_n) \int_0^1 [\mathrm{Fo}_{rm} \Theta_1(X) + (\mathrm{Fo}_{rm} p_{in} + 1) \Theta_0(X)] \bar{k}(\mu_n X)\, dX + k_2(\mu_n) \int_0^{\mathrm{Fo}} \Psi_2(\mathrm{Fo}^*) \exp[p_{in}(\mathrm{Fo} - \mathrm{Fo}^*)]\, d\mathrm{Fo}^* \right\},$$

$$\bar{k}(\mu_n X) = X^{m-1} k(\mu_n X); \quad (22)$$

$\nu_{121} = \varepsilon\mathrm{Ko}$, $\nu_{221} = -1$, $\nu_{112} = 1$, $\nu_{212} = 0$, $\nu_{122} = -\varepsilon\mathrm{Ko}$, $\nu_{222} = 1$, $\delta_{ljin}(l, j = 1, 2;\ i = 1, 2, 3)$ are Eqs. (9)–(10) into which instead of p we have, respectively, substituted the roots p_{in}; Δ_{in} are derived from (11) by replacing p by p_{in} and by eliminating the i-th cofactor, which is equal to zero: $\Delta_{1n} = \mathrm{Fo}_{rm}(p_{1n} - p_{2n})(p_{1n} - p_{3n})$; $\Delta_{2n} = -\mathrm{Fo}_{rm}(p_{1n} - p_{2n})(p_{2n} - p_{3n})$; $\Delta_{3n} =$

Kernels of integral transformations and characteristic equations

	m	X	Kernel of integral transformations	$k_1(\mu_n)$	$k_2(\mu_n)$	Characteristic equation	η_m
Plate	1	$\frac{x}{R}$	$\cos\mu_n X$	1	$(-1)^n$	$\sin\mu_n = 0$ $\mu_n = n\pi\ (n = 1, 2, \ldots)$	1/3
Cylinder	2	$\frac{r}{R}$	$J_0(\mu_n X)$	$\frac{1}{J_0^2(\mu_n)}$	$\frac{1}{J_0(\mu_n)}$	$J_1(\mu_n) = 0$	1/2
Sphere	3	$\frac{r}{R}$	$\frac{\sin\mu_n X}{\mu_n X}$	$\frac{\mu_n^2}{\sin^2\mu_n}$	$\frac{\mu_n}{\sin\mu_n}$	$\mathrm{tg}\,\mu_n = \mu_n$	3/5

Note: R is half the thickness of the plate, and the external radii of the cylinder and the sphere are $0 \le x \le R$, $0 \le r \le R$.

$= Fo_{rm}(p_{1n} - p_{3n})(p_{2n} - p_{3n})$; $\Psi_1 = Ki_q(Fo) - (1 - \varepsilon)LuKoKi_m(Fo)$ and $\Psi_2 = LuKi_m(Fo)$.

When D = 0 we have $p_1 = 2a_1 - \alpha/3 < 0$ and the double root $p_0 = p_2 = p_3 = -(a_1 + \alpha/3) < 0$.

To find the original from the mapping of (7) and (8) we must use formula (18): here it is necessary to find the limits of the denominators $\psi_1(p) = Fo_{rm}(p - p_0)^2$ and $\psi_2(p) = Fo_{rm}(p - p_1)$, for the numerator $\varphi(p)$ and its derivative, respectively, as $p \to p_1$ and $p \to p_0$.

If we take into consideration that $\Psi_2(p) = Fo_{rm}$, while $\varphi'(p) = \delta'_{lj}\Phi_{jL} + \delta_{lj}\Phi_{jL}$, after appropriate calculations and transformations we have a common notation for the solution in the form of (19) and (20), but here the value of δ_{ljin} in formulas (21) and (22) are equal to

$$\delta_{11in} = Fo_{rm} p_{in}^2 + p_{in} + Lu\,\mu_n^2,\quad \delta_{12in} = -p_{in}\varepsilon Ko,$$
$$\delta_{21in} = Lu\,Pn\,\mu_n^2,\quad \delta_{22in} = p_{in} + \mu_n^2$$
$$(i = 1,\ 3;\ p_{in} = p_{1n}\ \text{or}\ p_{0n}),$$
$$\delta_{112n} = [Fo(p_{0n} - p_{1n}) - 1](Fo_{rm} p_{0n}^2 + p_{0n} + Lu\,\mu_n^2) + (2\,Fo_{rm} p_{0n} + 1)(p_{0n} - p_{1n}),$$
$$\delta_{122n} = -\varepsilon Ko[Fo(p_{0n} - p_{1n})p_{0n} - p_{1n}],$$
$$\delta_{212n} = [Fo(p_{0n} - p_{1n}) - 1]\,Lu\,Pn\,\mu_n^2,$$
$$\delta_{222n} = [Fo(p_{0n} - p_{1n}) - 1](p_{0n} + \mu_n^2) + p_{0n} - p_{1n}.$$

The notation of expressions (21) and (22) remains without change, with the exception of the third term (i = 3):

$$\Phi_{l13} = 2\sum_{n=1}^{\infty}\delta_{l13n}\Delta_{3n}^{-1}k_2(\mu_n)k(\mu_n X)\times \int_0^{Fo}\Psi_1(Fo^*)(Fo - Fo^*)\exp[p_{0n}(Fo - Fo^*)]\,dFo^*,$$

$$\Phi_{l23} = 2\sum_{n=1}^{\infty}\delta_{l23n}\Delta_{3n}^{-1}k(\mu_n X)\Big[k_1(\mu_n)\exp(p_{0n}Fo)Fo_{rm}\times \int_0^1\Theta_0(X)\bar{k}(\mu_n X)\,dX + k_2(\mu_n)\times \int_0^{Fo}\Psi_2(Fo^*)(Fo - Fo^*)\exp[p_{0n}(Fo - Fo^*)]\,dFo^*\Big].$$

The denominators

$$\Delta_{1n} = \Delta_{2n} = Fo_{rm}(p_{1n} - p_{0n})^2,$$
$$\Delta_{3n} = -Fo_{rm}(p_{1n} - p_{0n}).$$

If D > 0, then $p_2 = x + iy$, $p_3 = x - iy$, where $x = -1/2(a_1 + b_1) - \alpha/3$ and $y = (3)^{1/2}/2(a_1 - b_1)$.

After substitution of the values of p_2 and p_3 and after having eliminated the imaginary parts of these complex roots, we have solutions (19) and (20), in which the first terms Φ_{lj1} for $p = p_1$ are written without change, with the exception of the denominator

$$\Delta_n = \Delta_{1n} = Fo_{rm}(p_{1n}^2 - 2x_n p_{1n} + x_n^2 + y_n^2) = 3Fo_{rm}(a_1^2 + a_1 b_1 + b_1^2).$$

If we take into consideration that when $\mu_0 = 0$, according to (13)–(15), x = y = 0, the following terms are written in the form:

$$\Phi^*_{lk} = \Phi^0_{lk} + 2\sum_{n=1}^{\infty}k(\mu_n X)\Delta_n^{-1}\times\{\exp(x_n Fo)z_k(y Fo)k_1(\mu_n)\times \int_0^1[\alpha_{lk}T_0 + \beta_{lk}\Theta_0 + Fo_{rm}\gamma_{lk}\Theta_1]\bar{k}(\mu_n X)\,dX + k_2(\mu_n)\int_0^{Fo}[\alpha_{lk}\psi_1(Fo^*) + \gamma_{lk}\psi_2(Fo^*)]\times \exp[X_n(Fo - Fo^*)]z_k[y_n(Fo - Fo^*)]\,dFo^*\},\quad l,\ k = 1,\ 2$$

where

$$\Phi^*_{11} = \Phi_{112} + \Phi_{122};\ \Phi^*_{12} = \Phi_{113} + \Phi_{123};$$
$$\Phi^*_{21} = \Phi_{212} + \Phi_{222};\ \Phi^*_{22} = \Phi_{213} + \Phi_{223};\ \Phi^0_{11} = \Phi^0_{21} = 0;$$
$$\Phi^0_{12} = m\int_0^1 X^{m-1}[T_0(X) - \varepsilon Ko\,Fo_{rm}\Theta_1(X)]\,dX + m\int_0^{Fo}[\Psi_1(Fo^*) - \varepsilon Ko\,\Psi_2(Fo^*)]\,dFo^*;$$
$$\Phi^0_{22} = m\int_0^1 X^{m-1}[\Theta_0(X) + Fo_{rm}\Theta_1(X)]\,dX + m\int_0^{Fo}\Psi_2(Fo^*)\,dFo^*;$$
$$z_1(y\,Fo) = \sin y\,Fo;\ z_2(y\,Fo) = \cos y\,Fo;$$
$$\bar{k}(\mu_n X) = X^{m-1}k(\mu_n X);$$
$$\alpha_{11} = (2\,Fo_{rm}x_n + 1)y_n - y_n^{-1}(p_{1n} - x_n)\times[Fo_{rm}(x_n^2 - y_n^2) + x_n + Lu\,\mu_n^2];$$
$$\beta_{11} = -\varepsilon Ko\,Lu\,\mu_n^2 y_n^{-1}(p_{1n} - x_n);$$
$$\gamma_{11} = -\varepsilon Ko[y_n - y_n^{-1}x_n(p_{1n} - x_n)];$$
$$\alpha_{12} = -[(2\,Fo_{rm}x_n + 1)(p_{1n} - x_n) + Fo_{rm}(x_n^2 - y_n^2) + x_n + Lu\,\mu_n^2];$$
$$\beta_{12} = -\varepsilon Ko(Lu\,\mu_n^2 - 2x^2 Fo_{rm});\ \gamma_{12} = p_{1n}\varepsilon Ko;$$
$$\alpha_{21} = -Lu\,Pn\,\mu_n^2(p_{1n} - x_n)y_n^{-1};$$
$$\beta_{21} = [Fo_{rm}(2x_n + \mu_n^2) + 1]y_n - y_n^{-1}(p_{1n} - x_n)\times\{\varepsilon Ko\,Lu\,Pn\,\mu_n^2 - Fo_{rm}[(x_n^2 - y_n^2) + x_n\mu_n^2] + x_n + \mu_n^2\};$$
$$\gamma_{21} = y_n - y_n^{-1}(p_{1n} - x_n)(x_n + \mu_n^2);$$
$$\alpha_{22} = -Lu\,Pn\,\mu_n^2;$$
$$\beta_{22} = -\{Fo_{rm}[(2x_n + \mu_n^2)p_{1n} - x_n^2 - y_n^2] + p_{1n} + \mu_n^2 + \varepsilon Ko\,Lu\,Pn\,\mu_n^2\};\ \gamma_{22} = -(p_{1n} + \mu_n^2);$$
$$\Delta_n = \Delta_{1n} = \Delta_{2n} = \Delta_{3n}.$$

There is some interest in comparing solutions (19)–(22) with the earlier-derived solutions for the heat- and mass-transfer equations of the parabolic type [2, 7]. In the general case, for the given values of the transfer criteria we have to calculate the corresponding functions and compare the derived solutions. However, in a number of specific cases it is possible to evaluate the relaxation term $\mathrm{Fo}_{rm}\partial^2\Theta/\partial\mathrm{Fo}^2$ in Eq. (2). As an example, let us assume $T_0 = 0$, $\Theta_0 = 0$, $\Theta_1 = 0$; $\Psi_1 = \text{const}$, $\Psi_2 = \text{const}$; then solutions (19)–(20) for $p_1 \neq p_2 \neq p_3 < 0$ have the form

$$T(X,\ \mathrm{Fo}) = m(\mathrm{Ki}_q - \mathrm{Ko\,Lu\,Ki}_m)\mathrm{Fo} + m\varepsilon\,\mathrm{Ko\,Lu\,Ki}_m\,\mathrm{Fo}_{rm} \times [1 - \exp(-\mathrm{Fo}/\mathrm{Fo}_{rm})] - \frac{1}{2}(\eta_m - X^2)(\mathrm{Ki}_q - \mathrm{NKi}_m) - \Lambda_1, \quad (23)$$

$$\Theta(X,\ \mathrm{Fo}) = m\,\mathrm{Ki}_m\,\mathrm{Lu}\,\{\mathrm{Fo} - \mathrm{Fo}_{rm}[1 - \exp(-\mathrm{Fo}/\mathrm{Fo}_{rm})]\} - \frac{1}{2}(\eta_m - X^2)[\mathrm{Pn\,Ki}_q + (1 - \mathrm{Pn\,N})\mathrm{Ki}_m] - \Lambda_2, \quad (24)$$

where

$$\Lambda_j = 2\sum_{n=1}^{\infty}\sum_{i=1}^{3} C_{ji}\,k(\mu_n X)\,k_2(\mu_n)\exp(p_{in}\mathrm{Fo})/p_{in}\Delta_{in} \quad (j = 1,\ 2);$$

$$C_{1i} = \delta_{11i}\Psi_1 + \delta_{12i}\Psi_2 = (\mathrm{Fo}_{rm}\,p_{in}^2 + \mathrm{Lu}\,\mu_n^2) \times (\mathrm{Ki}_q - \mathrm{N\,Ki}_m) + p_{in}(\mathrm{Ki}_q - \mathrm{Ko\,Lu\,Ki}_m);$$

$$C_{2i} = \delta_{21i}\Psi_1 + \delta_{22i}\Psi_2 = \mathrm{Lu}\,\{[\mathrm{Pn\,Ki}_q + (1 - \mathrm{Pn\,N})\,\mathrm{Ki}_m]\,\mu_n^2 + p_{in}\,\mathrm{Ki}_m\};$$

$$N = (1 - \varepsilon)\,\mathrm{Lu\,Ko},$$

η_m are given in the table.

As $\mathrm{Fo} \to \infty$, $\Lambda_j \to 0$ and $\exp(-\mathrm{Fo}/\mathrm{Fo}_{rm}) \to 0$ the values of the dimensionless mass-transfer potential Θ become equivalent to the solutions cited in [2] (page 177), but here we must introduce the criterion $\mathrm{Fo}' = \mathrm{Fo} - \mathrm{Fo}_{rm}$. The development of the mass-content field is somehow delayed by the dimensionless time Fo_{rm}. This indicates that the calculation of the mass transfer without consideration of Fo_{rm} leads to values that are exaggerated over the values actually observed. This is particularly significant in the moisture transport of rheological (structured) liquids. For this same reason there is also a slight change in the temperature field by the magnitude of the second term in Eq. (23), which may be rewritten to the form

$$T(X,\ \mathrm{Fo}) \simeq m\left\{(\mathrm{Ki}_q - \mathrm{NKi}_m)\left[\mathrm{Fo} - \frac{1}{2}(\eta_m - X^2)\right] - \varepsilon\,\mathrm{Ko\,Lu\,Ki}_m(\mathrm{Fo} - \mathrm{Fo}_{rm})\right\}. \quad (25)$$

The noted effect of relaxation time (or Fo_{rm}) on the process of capillary-diffusion moisture transport agrees with the conclusions drawn in references [1, 8].

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12 January 1967 Polytechnic Institute, Kalinin